

Nonlinearities  
in  
Reduced Order Modeling  
*- Workshop -*

---



**ETH** zürich

Zürich, 23 September 2016

ETH Zürich  
Department of Mechanical and Process Engineering  
Leonhardstrasse 21 – 8092 Zürich  
LEE Building  
Room LEE E 101

# FINAL PROGRAM

8:30-9:00	<b>Welcome Coffee</b>
<b>Oral Presentations</b>	
9:00-9:30	Adam Brink <i>On Modeling of Jointed Structures</i>
9:30-10:00	Wolfgang Witteveen <i>Accurate and computational efficient consideration of lap joint contact and friction by the use of modal derivatives</i>
10:00-10:30	Chiara Gastaldi <i>Reduced Order Models of Structures with Localized Nonlinearities</i>
10:30-11:00	Matt Allen <i>Nonlinear Normal Modes for Analysis of Geometrically Nonlinear Structures</i>
11:00-11:30	<b>Coffee break</b>
11:30-12:00	Daniel Rixen, Johannes Rutzmoser <i>Model order reduction for nonlinear dynamics models: a path in the jungle</i>
12:00-12:30	Paolo Tiso, Shobhit Jain <i>Hyper-reduction methods for nonlinear structural dynamics</i>
12:30-13:00	Xuanang Liu <i>Modeling the phenomenon of natural frequency shifting using reduced order backbone curve model</i>
13:00-14:30	<b>Lunch – Dozenterfoyer, ETH Hauptgebäude</b>
14:30-15:00	George Haller <i>Exact Model Reduction from a Generalized Slow-Fast Decomposition of Nonlinear Oscillations</i>
15:00-15:30	Robert Szalai <i>Reduced Order Models of Piecewise Smooth Systems</i>
15:30-16:00	Malte Krack <i>Nonlinear Modes and Their Suitability for Model Order Reduction of Mechanical Systems</i>
16:00-17:00	<b>Coffee break/Poster session</b>
17:00-18:00	<b>Plenary discussion</b>

## ABSTRACTS

### **On Modeling of Jointed Structures**

*Adam Brink, Sandia National Laboratories*

Certain industrial and defense applications require their structures to be extremely reliable, where failures could result in massive loss of life. These types of applications require a sound understanding of the structures that is not provided by traditional linear structural analyses. Additionally, quantifying the uncertainty of the structure, which is essential, often carries computational expenses that are beyond the realm of possibility for use with explicit nonlinear analyses. This talk will present how nonlinear reduced order models are utilized in these industries to address analysis of critical structures. The most common NROMs available at Sandia National Laboratories will be explored in addition to the barriers to their more frequent use.

### **Accurate and computational efficient consideration of lap joint contact and friction by the use of modal derivatives**

*Wolfgang Witteveen, Fachhochschule Oberösterreich*

The nonlinear contact and friction forces inside lap joints can significantly influence the dynamic behavior of flexible structures. Therefore, it is important to accurately compute the local (and very small) relative deformations inside the joint in order to get a realistic representation of the nonlinear forces. Common reduction basis alone, e.g. Component Mode Synthesis, are not suitable to capture these local deformations accurately, and therefore extended with problem-oriented trial vectors. The computation of these "joint trial vectors" is based on modal derivatives. In this talk, the theory of the "joint trial vectors" together with numerical examples which confirm the methods accuracy and efficiency would be presented.

## **Reduced-Order Models of structures with localized nonlinearities**

*Chiara Gastaldi, Politecnico di Torino*

In the field of nonlinear structural dynamics, systems with localized nonlinearities are of particular practical interest. Nonlinearities can be localized when, for example, they involve only a subset of the degrees of freedom (DOFs) of the system, as in the case of joints, cracks and friction dampers. While well-known techniques such as the harmonic balance method and Craig-Bampton component mode synthesis have generally been employed to generate ROMs in the past, they do not reduce degrees of freedom at the interfaces. Therefore the dynamic analysis of structures with localized nonlinearities remains a computationally demanding task because of the large size of the models involved.

Thus a series of specialized techniques for the reduction of the contact interface DOFs as well as the remaining linear DOFs has been developed. The key idea of these approaches is to represent the spatial coherences in the system dynamics with a set of linear systems with specifically chosen boundary conditions at the contact interface. The methods proposed here stem from an ongoing collaboration between University of Michigan and Politecnico di Torino started in 2012. They explore RO modeling of structures with intermittent contacts with friction [1], and with friction dampers [2,3]. These methods are described and compared, highlighting the differences, strengths and limitations of each one. Latest improvements such as a self-contained error metric capable of evaluating the quality of the reduced-order solution not involving the comparison with the full-order solution are presented [3].

### *Bibliography*

- [1] S. Zucca, B. I. Epureanu, Bi-linear reduced-order models of structures with friction intermittent contacts, *Nonlinear Dyn*, DOI 10.1007/s11071-014-1363-8 (2014)
- [2] M. Mitra, S. Zucca, B. I. Epureanu, Adaptive Microslip Projection (AMP) for Reduction of Frictional and Contact Non-Linearities in Shrouded Blisks, *J. Comput. Nonlinear Dynam*, Vol. 11, No. 4, (2016).
- [3] C. Gastaldi, S. Zucca, B. I. Epureanu, Jacobian Projection for the Reduction of Friction Induced Nonlinearities, accepted at ISMA 2016

## **Nonlinear Normal Modes for Analysis of Geometrically Nonlinear Structures**

*Matthew S. Allen, University of Wisconsin*

Geometric nonlinearity is an important consideration when designing many structures, for example the skin panels for future hypersonic cruise vehicles where intense pressures and aerodynamic heating can cause the panels to vibrate in and out of buckled states. Highly flexible joined-wing aircraft, which are being sought for station keeping at high altitude, can also exhibit nonlinear dynamic phenomena. It may also be possible to add a nonlinear element to an otherwise linear structure in order to reduce vibration levels and increase its life, leading to quieter automobiles or more durable spacecraft. All of these applications are challenging because numerical response predictions are expensive and these nonlinear systems exhibit a large range of phenomena, each of which may require a specialized analysis technique. This work shows that tremendous insight can be gained into the dynamics of these types of nonlinear structures using undamped nonlinear modal analysis.

This presentation highlights advances in modeling for geometrically nonlinear structures and discusses how nonlinear modes can be used in analysis, design and testing. The work focuses on structures that are modeled in commercial finite element software and uses a non-intrusive approach in which a series of static loads are applied to the structure and a nonlinear Reduced Order Model (ROM) is fit to the load-displacement behavior. Nonlinear modes prove to be effective in discerning whether the reduced basis contains the fidelity needed to capture the dynamics of interest and in assuring that the loads are large enough to allow the ROM to be accurately computed. Nonlinear modes are also found to be intimately connected to the random response of the structure, which is the most important loading in many aerospace applications. These concepts are demonstrated by applying them to a variety of finite element models, showing that the nonlinear modes provide tremendous insight into the dynamics of the structure.

## **Model Reduction of non-linear Dynamic Models: a Path in the Jungle**

*Daniel Rixen<sup>1</sup>, Paolo Tiso<sup>2</sup>, Johannes Rutzmoser<sup>1</sup>*

*<sup>1</sup>TU München, <sup>2</sup>ETH Zürich*

Model Reduction techniques for linear structures have been developed since the 60ies and can be considered as fairly mature and established. For non-linear models however, reduction approaches are much more recent: several ideas have been proposed and it is hard to keep track of all the different concepts published. It is a jungle! In this presentation we try to give a structured overview of the different techniques that are currently used or under investigation, starting from the well-known linear reduction techniques and explaining how they can be extended to non-linear systems.

## **Hyper-reduction methods for nonlinear structural dynamics**

*Paolo Tiso, Shobhit Jain*

*ETH Zürich*

We present a variant of the Energy Conserving Sampling and Weighting (ECSW) method that minimizes the offline cost associated to the construction of reduced order models of FE discretized geometrically nonlinear structural dynamics problems. The training sets required by ECSW are obtained by lifting linear modal analysis responses on a quadratic manifold generated with modal derivatives. The resorting on expensive POD of the full response is completely avoided.

## **Modelling the phenomenon of natural frequency shifting using a nonlinear reduced-order backbone curve model**

*X. Liu<sup>1</sup>, D. J. Wagg<sup>1</sup> and S. A. Neild<sup>2</sup>*

*<sup>1</sup>University of Sheffield, <sup>2</sup>University of Bristol*

In this paper, the phenomenon of natural frequencies shifting due to the nonlinear stiffness is modeled using a nonlinear reduced order model based on backbone curves. The structure in question is a rectangular plate with the ideal pinned constraint along all edges. To analytically explore the frequency shifting phenomenon, a four-nonlinear-mode based reduced-order model which contains both single-mode and mix-modes nonlinear terms is developed. The process of deriving the reduced order model is based on a normal form transformation, combined with a Galerkin type decomposition of the governing partial differential equation for the plate. This allows a low number of ordinary differential equations to be obtained, which in turn can be used to derive backbone curves that relate directly to the nonlinear normal modes (NNMs). The frequency shifting is then investigated relative to the backbone curves. Modal interactions, caused by nonlinear coupling terms are shown to cause the frequency shifts. Forced responses are computed using Computational Continuation Core (COCO) to validate the backbone curve results. Results for random excitation are also shown. An attempt is made to quantify the frequency shifting due to different nonlinear effects.

## **Exact Model Reduction from a Generalized Slow-Fast Decomposition of Nonlinear Oscillations**

*George Haller, ETH Zürich*

We establish general conditions under which a nonlinear mechanical system can be exactly reduced to a lower-dimensional model system on a slow manifold. Our Slow-Fast Decomposition (SFD) applies to any multi-degree-of-freedom, nonlinear, and possibly externally forced mechanical system, revealing parameter regimes in which a global reduced-order model exists for the system. The SFD conditions also turn out to be necessary for a reduced order model to exist, if we require the full nonlinear vibrations to synchronize exponentially fast in time with the reduced model. We obtain an expression for the domain boundary beyond which the reduced model loses its stability due to the loss of stability of the slow manifold. As a side result, SFD justifies the formal method of modal derivatives and Guyan reduction. We illustrate all these results on several mechanical examples.

## **Reduced order models of piecewise-smooth systems**

*Robert Szalai, University of Bristol*

In practical applications, piecewise-smooth models are already reduced. In mechanics, dry friction is a simplified model, which neglects to model precise surface physics. In biological applications switches are used to represent the open probabilities of ion channels. In electronics the steep exponential characteristics of diodes are replaced with discontinuous functions. At the same time the underlying dynamics, which may be mechanical, electrical or biochemical, is also reduced to lumped parameter models, using rigid bodies, reaction-kinetic equations or discrete electronic components. It is common practice to couple lumped parameter models with discontinuities, which then gives rise to standard piecewise-smooth systems.

The theory of piecewise-smooth systems is extensive and fraught with complications. The complications include non-uniqueness of solutions, non-existence of the centre manifold, combinatoric explosion of bifurcations as the underlying system dimension increases.

The talk therefore focusses on taking a different approach to model reduction of piecewise-smooth systems. Instead of performing model reduction independently on the smooth and non-smooth components of the system, we treat continuum models of piecewise-smooth systems as a single entity. This approach allows us to keep important ingredients of the dynamics that would otherwise be neglected. The result is a corrected reduced order model. We show that this model - under general conditions - has unique solutions and behaves more like a smooth system. The conventional piecewise-smooth model is generally a limit of the corrected and reduced model. For example, rigid body dynamics is recovered when all elastic wave speeds tend to infinity.

## **Nonlinear Modes and Their Suitability for Model Order Reduction of Mechanical Systems**

*Malte Krack, Universität Stuttgart*

The concept of nonlinear modes is an attempt to generalize the ideas of linear normal modes of vibration to nonlinear mechanical systems. In contrast to linear modes, nonlinear modes inherently capture the amplitude-dependent nature of the vibrational deflection form. Hence, they contain information on all relevant modes in a particular dynamical regime, so that a much smaller number of modes is typically sufficient to accurately reproduce the dynamical behavior of a given high-fidelity model. Owing to recent research efforts, broadly-applicable and efficient numerical methods are now available for nonlinear modal analysis and synthesis. Important extensions permit the treatment of nonsmooth and/or dissipative nonlinearities. This presentation will give an overview of the different concepts and numerical methods, and application examples will be provided in order to demonstrate current opportunities and limitations of the methodology. Applications range from systems with two-degree-of-freedom systems with geometric nonlinearity to turbomachinery bladed disks subject to dynamical contact interactions in joints.

## POSTERS

<p>Shobhit Jain, ETH Zürich  <i>Extended Energy Conserving Sampling and Weighting: Hyper-reduction over a nonlinear manifold for large nonlinear finite element discretized systems</i></p>
<p>Shobhit Jain, ETH Zürich  <i>Application of Global and Local Model Reduction on a Beam</i></p>
<p>Sten Ponsioen, ETH Zürich  <i>Applications of Spectral Submanifolds in Nonlinear Modal Analysis</i></p>
<p>Wolfgang Witteveen, Fachhochschule Oberösterreich  <i>Fast evaluation of research ideas related to Multibody Dynamics</i></p>
<p>Daniel Stadlmayr, Fachhochschule Oberösterreich  <i>Degree of freedom and constraint reduction for multibody systems</i></p>
<p>Chiara Gastaldi, Politecnico di Torino  <i>Reduced Order Modeling for Multistage Coupling of Cyclic Symmetric Structures</i></p>
<p>Giuseppe Battiato, Politecnico di Torino  <i>Reduced Order Modeling for Multistage Coupling of Cyclic Symmetric Structures with Friction at the Flange Joint</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>Comparison of Nonlinear System Identification Methods for Free Decay Measurements with Application to MEMS Devices</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>Sensing and Rating of Vehicle - Railroad Bridge Collision</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>Evaluation of Interface Reductions for Craig Bampton Substructured Models</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>Experimental Assessment of The Influence of Interface Geometries on Structural Response</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>A Numerical Round Robin to Predict the Dynamics of an Experimentally-Measured Brake-Reuss Beam</i></p>
<p>NOMAD Institute<sup>(o)</sup>  <i>Effects of the Far-Field Structure on Measurements of Joint Properties</i></p>

(o) The Nonlinear Mechanics and Dynamics (NOMAD) Research Institute is hosted by Sandia National Laboratories in Albuquerque, NM, for six weeks every summer. The goal of NOMAD is to train a new generation of researchers in modeling and measuring systems with strong nonlinearities, to develop professional networks for young researchers to rely upon, and to begin to address some of the challenging issues at the forefront of nonlinear mechanics and nonlinear dynamics that are too large for single institutions to solve on their own. The 2016 NOMAD Research Institute featured 25 graduate students and postdocs, two undergraduates, 11 high school students, and 30 mentors spanning 28 different institutions of higher education and industry in 17 different countries. Over the past three years, NOMAD has focused on jointed structures – both modeling and measuring their sensitivity to changes in geometry and test conditions. Several of the projects from the 2016 NOMAD Research Institute are being presented at the NROM workshop. For more information about the 2017 research institutes (both NOMAD and the Joints Institute), contact Matthew Brake (brake@rice.edu)

## HOW TO REACH THE VENUE

From the airport:

the ETH LEE building can be easily reached by taking tram 10 (direction Zürich, Bahnhofplatz/HB) just outside the airport, and step out at ETH/Universitätsspital. The trip takes 29 minutes. The LEE building is just few steps away from the tram stop.

Alternatively, the Zürich main station (Zürich HB) can be reached from the airport by train, and then the LEE building is within 5 minutes walking distance.

More information at [www.sbb.ch](http://www.sbb.ch).

